COMMUTATIVE ALGEBRA

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09. Minimal free resolution of finitely generated modules over local rings

PROPOSITION 9.1. *Let A be a local ring with the residue field k. Let M, N be A-modules. We assume N is a finite module. Then an A-module homomorphism $f : M \to N$ is surjective if and only if $f \otimes 1_k : M \otimes_A k \to N \otimes_A k$ is surjective.*

DEFINITION 9.2. Let

$$0 \leftarrow M \leftarrow F_0 \leftarrow F_1 \leftarrow F_2 \leftarrow \dots$$

be a free resolution. We say the resolution is **minimal** if

$$\mathrm{Ker}(d_i) \leftarrow F_i$$

is an isomorphism when tensored with k.

Let

$$0 \leftarrow M \leftarrow F_0 \leftarrow F_1 \leftarrow F_2 \leftarrow \dots$$

be a minimal free resolution. Then we have $\mathrm{Tor}_i^A(M, k) = \mathrm{rank}(F_k)$.