- 証明は道案内と同じ
- どうやってみつけたかは必要ない

[illegible] 6.11

(1) $\underbrace{(\mathrm{Image}\,P) + (\mathrm{Image}\,Q)}_{} \overset{?}{=} \mathbb{R}^n$

$\supseteq$

$$= \left\{ x + y \;\middle|\; \begin{array}{l} x \in \mathrm{Image}\,P \\ y \in \mathrm{Image}\,Q \end{array} \right\}$$

$$= \{ Pv + Qw \mid \cancel{u, v}\ v, w \in \mathbb{R}^n \}$$

A氏

$$\mathbb{R}^n = \overset{E_n}{\overbrace{(P+Q)}}\mathbb{R}^n = \underbrace{P\mathbb{R}^n}_{\mathrm{Image}\,P} + \underbrace{Q\mathbb{R}^n}_{\mathrm{Image}\,Q}$$

$\forall x \in \mathbb{R}^n$ に対して

$$x = E_n x = (P+Q)x$$

$$= \underbrace{Px}_{\in\,\mathrm{Image}\,P} + \underbrace{Qx}_{\in\,\mathrm{Image}\,Q} \in \mathrm{Image}\,P + \mathrm{Image}\,Q$$

故に $\mathbb{R}^n \subset \mathrm{Image}\,P + \mathrm{Image}\,Q$

(2) $\text{Image}\,P \cap \text{Image}\,Q = \{0\}$

$\forall x$ をとる

$x \in \text{Image}\,P$ より $x = Py$ $\quad (\exists y \in \mathbb{R}^n)$

$x \in \text{Image}\,Q$ より $x = Qz$ $\quad (\exists z \in \mathbb{R}^n)$

$\cancel{P^2 \neq I_n - Q}$

$P^2 = P$, $Q^2 = Q$, $PQ = O$

$$Px = P^2 y = Py = x$$

$$Px = PQz = Oz = 0$$

$\Rightarrow x = 0$

(3) $\mathrm{Image}(P) \perp \mathrm{Image}(Q) \Leftrightarrow {}^tP = P$

$\Rightarrow {}^\forall x, {}^\forall y \in \mathbb{R}^n$ に対して

${}^t x \, {}^t P Q y = 0$

$\Rightarrow {}^tPQ = 0$

$\Rightarrow {}^tP(1_n - P) = 0$

$\Rightarrow {}^tP = {}^tPP$ $\boxed{{}^t(AB) = {}^tB \, {}^tA}$

$\Rightarrow P \Rightarrow {}^tPP$

$\Rightarrow P = {}^tP$

No.

Date

$$\begin{bmatrix} a_{11} & & & 0 \\ & a_{22} & & \\ & & \ddots & \\ 0 & & & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{bmatrix} = A$$

Ae_1 Ae_2 Ae_3

$$A e_1 = 2 e_1$$

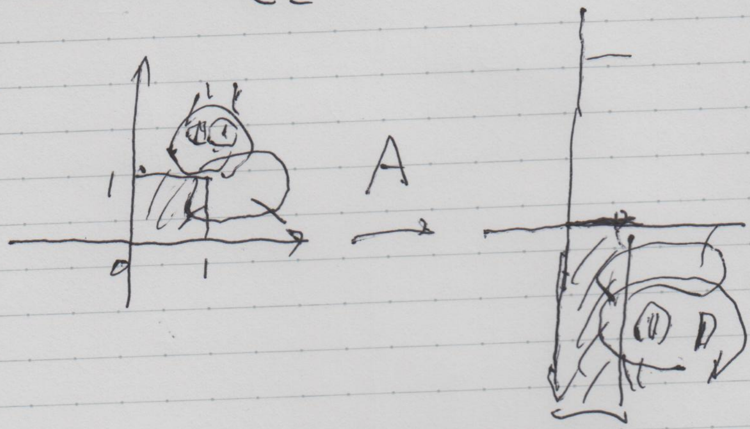
$$A e_2 = 3 e_2$$

$$A e_3 = -5 e_3$$

$$\begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix} \cdot \begin{bmatrix} b_1 & & 0 \\ & \ddots & \\ 0 & & b_n \end{bmatrix} = \begin{bmatrix} a_1 b_1 & & 0 \\ & \ddots & \\ 0 & & a_n b_n \end{bmatrix}$$

$$\overset{A}{\begin{bmatrix}1&0\\0&-4\end{bmatrix}}\underset{\substack{\parallel\\ e_1}}{\begin{bmatrix}1\\0\end{bmatrix}}=\begin{bmatrix}1\\0\end{bmatrix}=1\cdot e_1$$

$$\begin{bmatrix}1&0\\0&-4\end{bmatrix}\underset{\substack{\parallel\\ e_2}}{\begin{bmatrix}0\\1\end{bmatrix}}=\begin{bmatrix}0\\-4\end{bmatrix}=-4\,e_2$$



$$A=\begin{pmatrix}6 & 7\\ 8 & 5\end{pmatrix}$$

$$A\mathbb{v}=\lambda\mathbb{v} \qquad \mathbb{v}=\begin{pmatrix}x\\ y\end{pmatrix}$$

$$\begin{pmatrix}6-\lambda & 7\\ 8 & 5-\lambda\end{pmatrix}\mathbb{v}=\mathbf{0} \Leftarrow A\mathbb{v}-\lambda\mathbb{v}=\mathbf{0}$$

$$\boxed{(A-\lambda \underbrace{I_2}_{\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}})\,\mathbb{v}=\mathbf{0}}$$

$\updownarrow$

$$\begin{cases}(6-\lambda)x+7y=0\\ 8x+(5-\lambda)y=0\end{cases}$$

非自明な解をもつ

$\Updownarrow$

$$\det\begin{pmatrix}6-\lambda & 7\\ 8 & 5-\lambda\end{pmatrix}=0 \qquad \det(A-\lambda I_2)=0$$

Aの固有方程式，

(1) $\det(xE_2 - A)$

$= \det\left(\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} - \begin{pmatrix} 6 & 7 \\ 8 & 5 \end{pmatrix}\right)$

$= \det\begin{pmatrix} x-6 & -7 \\ -8 & x-5 \end{pmatrix}$ 行列式

$= (x-6)(x-5) - (-7)\cdot(-8)$

$= x^2 - 11x - 26$

(2) $x^2 - 11x - 26 = (x-13)(x+2)$

$13, -2$

(3) $A\begin{pmatrix} x \\ y \end{pmatrix} = 13\begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} -7 & 7 \\ 8 & -8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{cases} -7x + 7y = 0 \\ 8x - 8y = 0 \end{cases}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t \\ t \end{pmatrix} = t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\mathbb{C}$ $(\exists t \in \cancel{\mathbb{R}})$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(4)(改)

Aのもう一つの固有値 -2 に対する
Aの固有ベクトルをもとめよ.