AFFINGE GROUP SCHEMES 08

Let $\Bbbk$ be a ring. Let A be a $\Bbbk$-algebra. Let M be an A-bimodule. A k-linear map

$$D : A \to M$$

is called a derivation if

$$D(xy) = D(x)y + xD(y) \qquad (\forall x, \forall y \in A).$$

Proposition 0.1. *Let A be a $\Bbbk$-algebra. Let M be an A-bimodule. Let I be an ideal of A.*

(1) *M can be regarded as an A/I-module if $I.M = 0$ and $M.I = 0$.*
(2) *Let M be an A/I-bimodule. A derivation $D : A \to M$ can be factored through A/I if $D(I) = 0$.*
(3) *If $I = (f_1, f_2, \ldots, f_n)$, then $D(I) = 0$ if and only if $D(f_1) = 0, \ldots, D(f_n) = 0$.*

Proposition 0.2. *For any $\Bbbk$-algebra A, there exists a "universal derivation" $d : A \to \Omega_A$.*

Proposition 0.3. *Let A be a $\Bbbk$-algebra. Let M be an A-module. Let us denote by $A_\epsilon[M]$ the A-module $A \oplus M$ with the following multiplication:*

$$(a + \epsilon x)(b + \epsilon y) = ab + \epsilon(xb + ay)$$

Then for any $\Bbbk$-linear map $\varphi : A \to M$, an algebra homomorphism $x \mapsto x + \epsilon\varphi(x)$ is given if and only if φ is a derivation.