

AFFINGE GROUP SCHEMES 10

DEFINITION 0.1. Let G be an affine group scheme. We define the commutator group $[G, G]$ as a union

$$[G, G](\mathbb{k}) = \cup_n \{[x_1, y_1][x_2, y_2] \dots [x_n, y_n] \mid x_1, \dots, x_n \in G; y_1, \dots, y_n \in G\}$$

We define the derived group $\mathcal{D}G$ of G as the Zariski closure of the commutator group $[G, G]$. We further define inductively $D^k G = D(D^{k-1}G)$.

DEFINITION 0.2. An affine group scheme G is called solvable if $D^k G = \{e\}$ for some k .

EXAMPLE 0.3.

$$\{A \in \mathrm{GL}_n(\mathbb{k}); A \text{ is upper triangular}\}$$

is solvable.